Prediction of Slug Frequency in Horizontal Two-Phase Slug Flow

Available data on slug frequency in horizontal two-phase intermittent flow are predicted with adequate accuracy by assuming that the slug frequency is one half of the frequency of the unstable waves precursors of slugs, as determined according to published analyses of finite amplitude waves in conduits. The experimental effects of gas and liquid flow rates, pipe diameter, gas density and liquid viscosity on slug frequency are explained by modifications of the wave properties due to changes in the liquid level of the stratified flow existing in the pipe inlet region prior to slug formation. Simple generalized equations are provided to estimate the slug frequency for engineering calculations.

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Introduction

Understanding the mechanisms governing cocurrent gas-liquid flow in horizontal conduits is of great practical and industrial importance. This task is particularly difficult in the case of the slug (intermittent) flow regime, due to its peculiar unsteady and composite nature causing large variations in the space and time distributions of the two phases.

Considerable progress has been made in this respect during the last two decades. A comprehensive physical model for horizontal slug flow was first published by Dukler and Hubbard (1975), and later developed and extended by Nicholson et al. (1978). Predictions of pressure gradients and holdups generated by a modified form of the same model have been recently reported to be in good agreement with extensive experimental observations taken during oil-air two-phase flow in pipes of three different diameters (Kokal and Stanislav, 1989b).

To predict the detailed hydrodynamics of the intermittent flow, such mechanistic models require the following data, in addition to fluid properties, gas and liquid flow rates, and pipe geometry: the gas content in the body of the liquid slug, and the slug frequency or the slug length. A number of attempts have been made to arrive at a closure of the model equations and thus make it possible to predict all of the slug characteristics. A physical model for the gas holdup in slugs has been published by Barnea and Brauner (1985). On the other hand, no satisfactory model exists at present for the prediction of the slug frequency or of the slug length.

Slug frequency data have been reported by several authors (Gregory and Scott, 1969; Vermeulen and Ryan, 1971; Greskovich and Shrier, 1972; Dukler and Hubbard, 1975; Heywood and Richardson, 1979; Knervold et al., 1984; Kago et al., 1987), but attempts at correlating or predicting this variable have been scarce and unsuccessful. Gregory and Scott (1969) and Greskovich and Shrier (1972) published dimensional correlations of slug frequency data with essentially no physical basis. Taitel and Dukler (1977) have proposed the only available predictive model for slug frequency during gas-liquid flow in horizontal and near-horizontal pipes. These authors considered that the slug frequency is determined by the process of slug formation in the entrance region, where unstable waves growing on a stratified film eventually block the gas passage and generate a liquid slug. Immediately afterwards, the liquid-level drops; hydrostatic forces then rebuild the film to its original level, completing the cycle. Taitel and Dukler assumed that the slug frequency equals the inverse of the time interval needed to rebuild the film equilibrium level and developed unsteady, monodimensional conservation equations for mass and momentum in open channel flow in order to calculate the characteristic time for this process. They reported satisfactory agreement with the data of Hubbard (1965) and with those of Gregory and Scott (1969). The same authors outlined also a generalized representation of their theory, where the calculated nondimensional slug frequency is given in graphical form as a function of four dimensionless groups of variables in the case of horizontal flow.

The Taitel-Dukler analysis introduces the interesting concept that the slug frequency is determined by the mechanism of slug formation. It, however, suffers from a fundamental contradiction. In writing the equation of motion for the gas phase, the authors intentionally neglect the "Bernoulli-effect" term, which describes the formation of unstable waves, precursors of slugs, as a result of a reduction in pressure above the wave due to gas acceleration. This simplification prevents intractable difficulties in the numerical solution of the model equations. By so doing, however, Taitel and Dukler overlook the physical phenomenon governing the entire process of slug formation. Also, the authors arbitrarily identify the lower liquid level from which the film begins to rebuild (the initial condition for the liquid level in their solution), as the "stability level" or as the minimum level at which slug flow exists in the pipe.

A more practical problem with the Taitel-Dukler analysis is that they have published only one chart containing their dimensionless solution for slug frequency, covering a limited range of possible flow conditions. This has prevented a general validation of the model, particularly concerning its capability to predict the effects of the fluid properties. Only Knervold et al. (1984) have reported a ±15% agreement between the Taitel-Dukler predictions and three slug frequency data points measured in a 2.54-cm-ID pipe during intermittent flow of nitrogen and a kerosene-mineral oil mixture.

Nicholson et al. (1978) developed an alternative approach to the closure of the existing models for slug flow which avoids the estimation of slug frequency. It is based on the empirical observation that measured values of the slug length L_s in intermittent flow are relatively insensitive to changes in the gas and liquid flow rates, the average L_s being ≈ 30 pipe diameters in the Nicholson et al. experiments. Assuming such a constant value, satisfactory agreement was noted between flow predictions of the model and experimental data obtained for an air-oil system in a 5.12-cm-ID pipe.

The same method has been again proposed recently by Kokal and Stanislav (1989b). Barnea and Brauner (1985) provided this approach with some physical basis by relating the stable slug length to the distance needed to reestablish the turbulent velocity distribution in the slug body after it is perturbed by the liquid film entering its front. Simulating the process as a jet entering a large reservoir, they calculated a characteristic slug length of 32 pipe diameters. Developing the concept of boundary layer relaxation originally advanced by Maron et al. (1982), Dukler et al. (1985) extended the analysis by including the viscous effects due to the pipe wall and estimated the minimum stable slug length as the distance required for the full development of a momentum boundary layer along the slug body. A comparison with some measurements showed, however, that the length of stable slugs is greater than that predicted by this theory.

It is apparent that the use of the slug length to effect the closure of the slug flow models has no fundamental advantage on the use of the slug frequency, but perhaps for the reduced sensitivity of the model equations. However, the degree of randomness associated with the slug length data is greater (Dukler and Hubbard, 1975), and some physical insight is lost: Nicholson et al. (1978) demonstrated that a slug flow model relying on the assumption of a fixed value for L_s can estimate on the average the measured values of slug frequency, but cannot reproduce the complex maxima-minima behavior exhibited by such data upon varying the gas flow rate. Furthermore, the determination of slug frequency may be important not only with respect to its use in mechanistic slug flow models. For example, slugs determine dangerous vibrations in pipes, causing resonance to occur when the slug frequency approaches the eigenfrequency of the pipe system. Erosion-corrosion phenomena in pipelines may also be influenced by the frequency of slugs in intermittent flow (Knervold et al., 1984).

In this paper an analysis is presented for two-phase horizontal intermittent flow which relates the slug frequency to the conditions of flow in the pipe entry region, where mixing of gas and liquid phases occurs. It is assumed that the slug frequency is inversely proportional to the period of the unstable waves formed at the gas-liquid interface of the originally stratified flow, and such waves eventually grow to generate the liquid slugs. Predictions of slug frequency derived from this analysis compare successfully with the data covering a representative range of flow conditions, exhibit the same characteristic features of the data, and account for the observed effects of gas and liquid flow rates, pipe diameter and fluid properties.

Theory

The process of slug formation occurring in an horizontal conduit following injection of air in a flowing liquid stream has been described by several investigators. It is well established that slugs originate from unstable waves formed at the gas-liquid interface of a stratified flow, which eventually grow in amplitude to block the gas passage. The wave instability results from a "Bernoulli effect," which is responsible for a normal force component acting on the wave crest in the opposite direction of gravity. On the basis of an extensive photographic study, Kordyban (1985) has suggested that slugs form as a result of local instability at the wave crest rather than due to instability of the whole wave. While the analysis of wave instability has been used with success to model the flow regime transition between stratified and intermittent flow (Kordyban and Ranov, 1970; Wallis and Dobson, 1973; Taitel and Dukler, 1977; Kordyban, 1977a; Mishima and Ishii, 1980; Kokal and Stanislav, 1989a), no attempt has been made to investigate whether the features of the waves responsible for the slug formation correlates with characteristic properties of the slugs in fully-developed slug flow.

Since slugs are generated by waves growing to occlude the pipe, it may be reasonable to expect that the number of slugs formed per unit time be inversely dependent on the time interval required for the development of each unstable wave up to the point of pipe bridging, which is related to the wave period. In the present work it is proposed that the slug frequency ω, i.e., the frequency of passage of slugs past a fixed point downstream of the pipe entry, is inversely proportional to the period τ of the waves precursors of the slugs,

$$\omega = C_1/\tau \tag{1}$$

where τ is evaluated relative to a frame of reference moving at the average velocity of the liquid phase.

The relation between slug frequency and wave period underlying Eq. 1 is certainly rather complex. Two factors may contribute to make ω less than $1/\tau$:

a. Only a fraction of the waves may effectively form slugs.

b. Even after their generation, a part of the slugs may become unstable and be destroyed.

Concerning point a above, it has been actually observed that at the transition to slug flow only a rare wave forms a slug; in developed slug flow, however, essentially every wave bridges the pipe. On the other hand, it is well known that the frequency of formation of slugs is generally greater than the slug frequency observed downstream of the pipe entrance (Taitel and Dukler, 1977; Dukler et al., 1985). In fact, immediately after a slug is formed, the liquid level drops, so that the trailing wave either dies off before it can bridge the pipe or generates an ill-fated slug which cannot sustain itself, and then is destroyed somewhere downstream, possibly by being incorporated into the following slug. The destruction of the wave or of the corresponding slug, however, contributes to restoring the original configuration of the stratified flow, and another stable slug can be generated. Dukler et al. (1985) have suggested that the observed slug length might typically result from overlapping of two originally distinct slugs.

Based on the above statement and the observation that a considerable degree of randomness is involved in the processes of slug formation and slug overlapping, we shall assume that on the average (in a statistical sense) each wave generates a slug, but only half of the slugs survive as single entities while traveling down the pipe. Accordingly, we shall make use of $C_1 = 1/2$ in the following treatment. Determination of the slug frequency is thus related by Eq. 1 to the estimation of the wave period, τ, or of its reciprocal ν, i.e., the wave frequency. This problem is addressed in the following section.

Estimation of the wave frequency

The frequency of the waves ν, relative to a frame of reference moving with the liquid velocity, can be determined from the knowledge of their celerity, C, and wave number, k, as

$$\nu = k\frac{C}{2\pi} \tag{2}$$

Kordyban and Ranov (1970) and later Mishima and Ishii (1980) have published nonlinear analyses of the inviscid two-dimensional flow of air and water in a rectangular channel, where waves of finite amplitude are considered. When flow conditions occur corresponding to the slug flow regime, the waves become unstable: according to the above treatments, the wave velocity can be expressed in this case as (Mishima and Ishii, 1980)

$$C = \frac{\Phi_G(kh_G)}{\Phi_L}\frac{\rho_G}{\rho_L}u_G \tag{3}$$

In Eq. 3,

$$\Phi_G(kh_G) = k\eta \coth(kh_G - k\eta)[1 + \tfrac{1}{2}k\eta\coth(kh_G - k\eta)]$$

and

$$\Phi_L = k\eta(1 - \tfrac{1}{2}k\eta)$$

where η is the interface elevation above the equilibrium level. In the derivation of Eq. 3, it has been assumed that $\rho_G \ll \rho_L$, $u_G \gg u_L$, and $(kh_L + k\eta) \gg 1$; also, surface tension effects have been neglected. Both Mishima and Ishii and Kordyban and Ranov regard $k\eta$ essentially as a constant parameter. The former authors assume $k\eta = 1$, which corresponds to the limiting amplitude of deep waves, whereas Kordyban and Ranov make use of $k\eta = 0.9$, allowing for the curvature of the wave crest: this secures the best fit of their data for transition to slug flow. We shall also assume $k\eta = 0.9$ in the calculations of the present work.

Combining Eqs. 3 and 2, and simultaneously multiplying and dividing by h_G, the equilibrium gas depth of the undisturbed stratified flow, one obtains

$$\nu = \frac{1}{2\pi}\frac{kh_G\Phi_G(kh_G)}{\Phi_L}\frac{\rho_G}{\rho_L}\frac{u_G}{h_G} \tag{4}$$

The wave number k appearing in Eq. 4 can be determined following the concept that the slug is formed by the "most dangerous wave" (Mishima and Ishii, 1980), i.e., by imposing the growth rate of unstable waves to be maximum. The wave number for waves of maximum growth rates satisfies the condition

$$\frac{d}{dk}k|C_I| = 0 \tag{5}$$

where, as reported by Mishima and Ishii (1980),

$$|C_I| = \sqrt{\frac{g}{\Phi_L k} - \frac{\Phi_G}{\Phi_L}\frac{\rho_G}{\rho_L}u_G^2}$$

represents the imaginary part of the wave velocity when waves are unstable. At the onset of slugging, $|C_I| = 0$, which is the condition for neutrally-stable waves.

Rearrangement of Eq. 5 yields

$$2kh_G\Phi_G + (kh_G)^2\Phi_G' - \left(\frac{gh_G}{u_G^2}\frac{\rho_L}{\rho_G}\right)k\eta = 0 \tag{6}$$

with $\Phi_G' = d\Phi_G/d(kh_G)$.

Solution of Eq. 6 provides the dimensionless wave number kh_G corresponding to the maximum wave growth rate as a function of the group $F_G = [\rho_G u_G^2/(\rho_L g h_G)]^{0.5}$. Values of kh_G are plotted vs. F_G in Figure 1 (Curve B) for $k\eta = 0.9$. The figure presents also the values of kh_G at neutral stability (curve A) for $k\eta = 0.9$. The intersection between the two curves determines the condi-

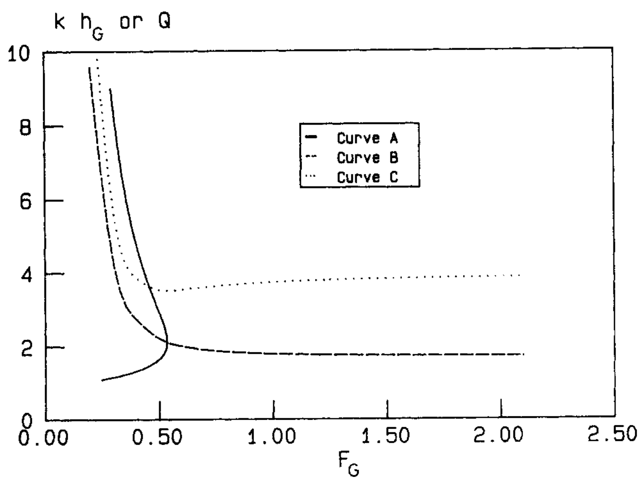

Figure 1. Values of kh_G or Q vs. F_G.
Curve A = dimensionless wave number kh_G at neutral stability
Curve B = dimensionless wave number kh_G at maximum wave growth rate, Eq. 6
Curve C = values of $Q = \frac{1}{2}kh_G\Phi_G/\Phi_L$ at maximum wave growth rate

tions for the onset of slug flow: this occurs at about $F^*_G = 0.53$ in Figure 1 with $k\eta = 0.9$, while Mishima and Ishii find a critical $F^*_G = 0.49$ with $k\eta = 1$. In both cases, such results are in excellent agreement with the experimental findings of Wallis and Dobson (1973). These authors correlated successfully the data of Kordyban and Ranov (1970) as well as their own data on the onset of slugging for flow of air over water, covering a wide range of changes in geometrical variables, by the semiempirical relationship $F^*_G = 0.5$.

Notably, the assumption of $k\eta = 0.9$ corresponds to a wave height-to-length ratio of 0.286, which is higher than that observed by Kordyban (1977b, 1985). It is possible that, in incipient slug flow, bridging of the pipe and slug formation occurs before unstable waves reach their limiting steepness. Using $k\eta = 1$, however, Mishima and Ishii not only generated good predictions for the onset of wave instability at the transition to slug flow, as already mentioned, but also reported satisfactory agreement with measured wave lengths. Thus, the theoretical analysis of wave instability adopted for the purposes of this work seems consistent with experimental observations of wave features relevant in determining slug flow characteristics.

In Figure 1, curve C shows also a plot of the function

$$Q = \frac{1}{2}\frac{kh_G\Phi_G(kh_G)}{\Phi_L}$$

against F_G at maximum wave growth rate, evaluation of this function being required by Eq. 4 in order to estimate the wave frequency. It is apparent that Q is a weak function of F_G. On assuming an average value in the range $0.5 \leq F_G \leq 2.1$,

$$Q = \frac{1}{2}\frac{kh_G\Phi_G(kh_G)}{\Phi_L} \approx 3.80 \tag{7}$$

Combining Eqs. 1, 4 and 7, and introducing $C_1 = 1/2$, we obtain eventually

$$\omega = 0.61\,\frac{\rho_G}{\rho_L}\frac{u_G}{h_G} \tag{8}$$

In order to use Eq. 8 for prediction of the slug frequency, it is still necessary to estimate h_G, the equilibrium gas depth, the actual gas velocity u_G being also a function of h_G. Here h_G refers to the stratified flow which establishes in the pipe entry region, i.e., the stretch of pipe included between the pipe inlet and the point of slug formation. Prediction of ω is thus reduced to the problem of determining the configuration of the inlet stratified flow.

Estimation of the equilibrium gas depth in the entrance stratified flow

Among the available analyses of stratified flow, the simplified treatment presented by Taitel and Dukler (1976a,b) for circular pipes has gained a vast popularity. These authors show that the equilibrium level is uniquely a function of the Lockhart-Martinelli parameter (Lockhart and Martinelli, 1949).

$$X = \left[\frac{(-dP/dx)_{SL}}{(-dP/dx)_{SG}}\right]^{0.5}$$

provided that f_i/f_G, the ratio of the interfacial to the wall gas friction factor, can be regarded as constant. In fact, by equating the pressure gradient $(-dP/dx)$ in both phases, Taitel and Dukler find in dimensionless form

$$X^2 = \frac{[\overline{S}_G/\overline{A}_G + (\overline{S}_i/\overline{A}_L + \overline{S}_i/\overline{A}_G)f_i/f_G](\overline{u}_G\overline{D}_G)^{-m}\overline{A}_L}{(\overline{u}_L\overline{D}_L)^{-n}\overline{S}_L}\left(\frac{\overline{u}_G}{\overline{u}_L}\right)^2 \tag{9}$$

where m and n are the exponents of Blasius-type expressions for the friction factors of the gas and liquid, respectively:

$$f_G = C_G Re_G^{-m}; \quad f_L = C_L Re_L^{-n}$$

Based on the geometry of stratified flow in circular pipes, it can be shown that all the variables appearing in the righthand side of Eq. 9 depend only on $\overline{h}_G = h_G/D$ (Govier and Aziz, 1972, p. 563). Hence, if f_i/f_G = constant, Eq. 9 represents an implicit equation for $\overline{h}_G$ as a function of X alone.

In applying the Taitel–Dukler approach to the estimation of h_G, however, a problem associated with the evaluation of the interfacial friction factor f_i arises. In their original paper, the authors report that values of two-phase pressure drops and liquid holdups calculated from their theory with $f_i/f_G = 1$ compare satisfactorily with experimental data. Nevertheless, the assumption that the interfacial friction factor of gas, f_i, is equal to the wall friction factor, f_G, seems applicable only to a smooth interface. The gas-liquid interface is often perturbed by waves, especially when slug flow conditions are approached. In this case, f_i may be significantly greater than f_G. Shoham and Taitel (1984) presented a two-dimensional model for stratified turbulent-turbulent gas-liquid flow which approximately takes into account the interfacial structure by assuming that it acts as a fully-developed rough surface. Better agreement with experimental data than obtained with the original Taitel-Dukler theory was claimed by the authors. Andritsos and Hanratty (1987) have reported interfacial stresses calculated from measurements of liquid height and pressure drop for fully-developed horizontal stratified flow. Values for f_i/f_G as high as 15 resulted from their data corresponding to values of the film thickness $h_L/D = 1 - h_G/D$ between 0.03 and 0.22. A semiempirical design equation was derived, yielding f_i/f_G as a function of the superficial gas velocity u_{SG} and of h_L/D. However, extrapolation of this relation to the stratified flow existing prior to slug formation, where $h_L/D > 0.35$ (Barnea et al., 1982), provides unrealistically high values of f_i/f_G.

In order to allow for the effects of a wavy interface without giving up the simplicity of the Taitel-Dukler approach, in the present work it is assumed the $f_i/f_G = 1$ when the gas phase is in laminar flow, whereas $f_i/f_G = 2$ in turbulent gas flow conditions. This treatment relies on the assumption that the wavy liquid surface acts as a rough wall for turbulent gas flow, whereas the interfacial friction factor is independent of surface roughness when the gas flow is laminar. The "equivalent roughness" approach has been applied in the literature to the analysis of the interfacial shear stress in gas-liquid stratified flow (Lilleleht and Hanratty, 1961). The influence of an increasing gas velocity on f_i has been empirically incorporated by allowing it to become twice the friction factor for a smooth surface. For typical situations considered in this work, the latter assumption results in f_i-values close to the value ($f_i = 0.0142$) adopted by Shoham and Taitel (1984) in evaluating the interfacial shear stress for air/water stratified flow, based on measurements by Cohen and Hanratty (1968).

Results and Discussion

Comparison with experimental data

Published slug frequency data measured by several authors with different methods for various gas/liquid systems are compared in this section with slug frequencies predicted by Eq. 8.

The available data involve laminar, transitional and turbulent flow of the gas phase in the entry stratified flow, and turbulent or laminar flow of the liquid phase. For the purpose of estimating $\bar{h}_G$, it is necessary to define boundaries between the flow regimes. In stratified gas-liquid flow, the gas phase reportedly becomes turbulent when the superficial gas Reynolds number Re_{SG}, defined as $(\rho_G u_{SG} D/\mu_G)$, exceeds 5,000 (Govier and Aziz, 1972, p. 566). It seems more appropriate, however, that the effective Reynolds number $Re_G = (\rho_G u_G D_G/\mu_G)$, based on the actual gas velocity and on the gas hydraulic diameter, should control the flow regime transitions. For the range of pipe diameters and fluid properties considered, $Re_{SG} = 5{,}000$ corresponds approximately to a value of 8,000 for Re_G. This criterion has been chosen to locate the boundary of the turbulent flow regime, whereas the laminar regime is assumed to exist at $Re_G <$ 2,500. Table 1 lists the equations employed in the calculation of the wall and interface gas friction factors according to the prevailing flow regime.

To estimate the liquid friction factor, standard correlations for smooth tubes have been used,

$$f_l = 16/Re_L \qquad Re_{SL} \leq 1{,}000$$
$$f_l = 0.046\, Re_L^{-0.2} \qquad Re_{SL} > 1{,}000$$

It is worth noting that the fluid properties and the flow conditions involved in Eq. 8 refer to the entrance region of the conduit, where the slugs are formed. Accordingly, the gas velocity and the gas density at the pipe inlet should be used in the calculations. While the exact point of slug formation is unknown, in developed slug flow formation of slugs it is typically observed at a short distance from the pipe inlet. Within a good approximation, then, the gas velocity and the gas density at the pipe inlet can be used in the calculations. This in turn requires estimation of the pressure loss along the tubes. The semiempirical Lockhart-Martinelli-type correlation of Chisolm (1967) has been chosen for this purpose. Mechanistic models of slug flow (Dukler and Hubbard, 1975; Nicholson et al., 1978; Kokal and Stanislav, 1989b) could also be used; however, they require simultaneous solution of the model equations for the pressure gradient and of Eq. 8 for ω. Diagnostic computations have shown that, for the experimental flow configurations considered in this work, the effect of the pressure drop on the estimates of ω is weak, becoming relatively more significant only at the highest gas rates.

Table 1. Equations for Evaluating Gas Friction Factors at the Pipe Wall and Gas-Liquid Interface

	f_G	f_i/f_G
$Re_G \leq 2{,}500$	$16\, Re_G^{-1}$	1
$2{,}500 < Re_G \leq 8{,}000$	$1.98 \times 10^{-3}\, Re_G^{0.15}$	$9.124 \times 10^{-3}\, Re_G^{0.60}$
$Re_G > 8{,}000$	$0.046\, Re_G^{-0.2}$	2

Hubbard and Dukler's Data. Experimental slug frequency data obtained from visual slug counts were reported by Hubbard (1965) and by Dukler and Hubbard (1975) for flow of air and water in a 3.81-cm-dia. horizontal pipe. These data are compared in Figure 2 with results calculated according to Eq. 8 and represented by lines. In view of the predictive nature of the present theory, the agreement appears satisfactory both qualitatively and quantitatively, only the data points at the highest liquid superficial velocity lying slightly above the theoretical curve. The calculated predictions increase with increasing liquid rates and exhibit a characteristic minimum with increasing gas rates, as shown by the data.

Heywood and Richardson's Data. Heywood and Richardson (1979) published extensive slug frequency data measured by γ-ray absorption during the cocurrent flow of air and water in a 4.2-cm-dia. horizontal pipe. Figure 3 illustrates how some of their data compare with the predictions based on Eq. 8. The agreement is again satisfactory.

Gregory and Scott's Data. The data of Gregory and Scott (1969) refer to flow of carbon dioxide and water in a 1.9-cm pipe. Slug frequencies were measured both by visual slug counts and by counting the number of slug pressure pulses recorded over a known period of time. As shown in Figure 4, the quantitative agreement with the theory is less encouraging than that for the previous sets of data, even though the calculated slug frequencies still exhibit the correct trends. In the original paper, the data are plotted as ω vs. u_s, the measured slug velocity. In this work the corresponding value of the superficial gas velocity u_{SG}, required for the calculations, has been estimated for each data point from the correlation

$$u_S = 1.35\,(u_{SG} + u_{SL})$$

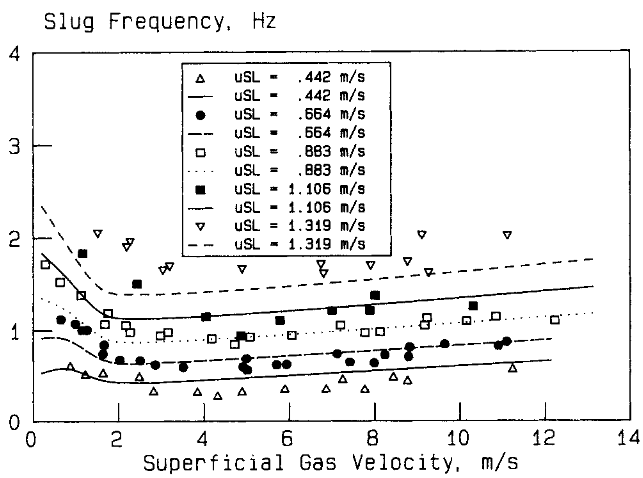

Figure 2. Experimental vs. calculated values of slug frequency for air/water, $D = 3.81$ cm.
Assumed equivalent length of test pipe, 20 m
Data of Dukler and Hubbard (1975)

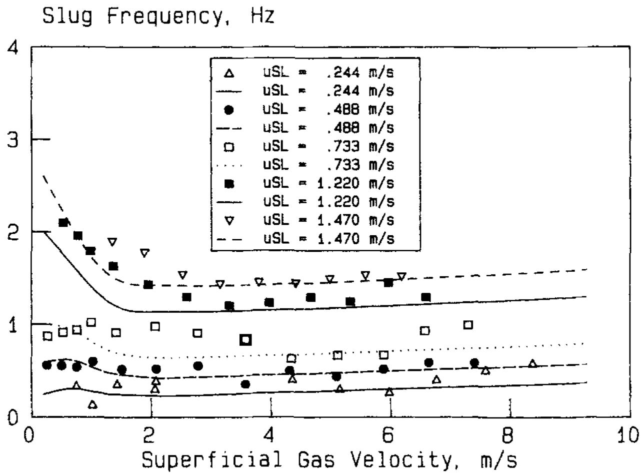

Figure 3. Experimental vs. calculated values of slug frequency for air/water, D = 4.2 cm.
Assumed equivalent length of test pipe, 18 m
Data of Heywood and Richardson (1979)

proposed by the same authors, which, however, involves significant scatter. Hence, greater inaccuracies are associated with this set of data than with those of Figures 2 and 3.

Data of Kago et al. Using a videotape recorder, Kago et al. (1987) measured slug frequencies in a horizontal 5.15-cm-ID, 25-m-long pipe during an investigation of the axial mixing of liquid in two-phase slug flow. The gas used was air, and the liquid phase was either water or an aqueous solution of polyethylene oxide (PEO), with viscosities of 1 and 38 mPa · s, respectively. The slug frequency was found to increase with increasing liquid viscosity. As shown in Figure 4, predictions obtained from Eq. 8 are in good agreement with the data corresponding to both the low and the high viscosity of the liquid phase. In the latter case, the liquid flow regime in the stratified entry flow is laminar.

Data of Knervold et al. Knervold et al. (1984) measured slug frequencies, along with slug lengths, slug velocities, and velocity distributions in the slug units, using optical probes. Their experimental system consisted of a mixture of mineral oil and kerosene flowing cocurrently with nitrogen in a 2.4-cm-ID horizontal pipe. The viscosity of the liquid was 12 mPa · s. The *five slug frequency data points reported by these authors are* compared in Figure 4 with predictions obtained from Eq. 8, assuming laminar liquid flow in the entry zone. The agreement is satisfactory, the maximum deviation being about −29% with an average error less than 10%. Notably, the values of L_S/D determined from the experimental slug lengths reported by Knervold et al. range between 15 and 22, differing significantly from the average value of 30 suggested by Nicholson et al. (1978).

Data of Vermeulen and Ryan. A comparison of the theoretical predictions was also attempted with the slug frequency data of Vermeulen and Ryan (1971), measured for air/water in a 1.27-cm-ID tube. In this case, however, it was found that the data were consistently overpredicted by Eq. 8, the average error being ≈ 50%. It is suspected that the failure of the present theory to accurately describe the frequencies of slugs generated in such a small pipe should be attributed to having neglected the influence of surface tension in the expression for the wave frequency. Such an influence, in fact, becomes more important in the case of small-diameter tubes.

The agreement between the present theory and all the sets of data considered is summarized in Table 2. When evaluating these results it may be worth recalling that the data were read from the figures of the original papers, a process which is expected to add ≈10% to the experimental uncertainty.

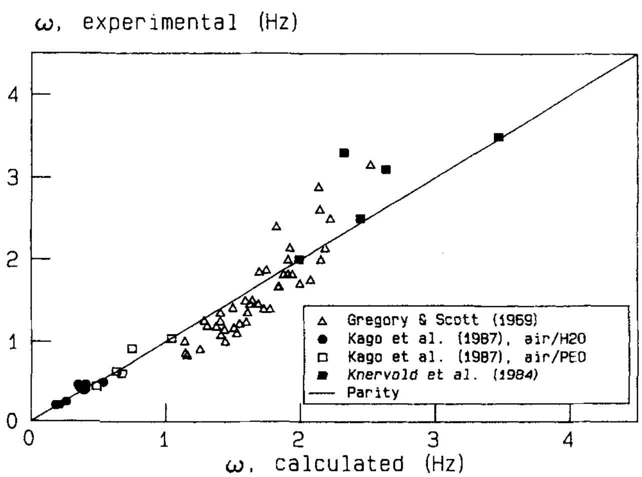

Figure 4. Experimental vs. calculated values of slug frequency from other data.
Data of Gregory and Scott (1969): CO_2/water, D = 1.9 cm, assumed equivalent length of test pipe = 5.7 m
Data of Kago et al. (1987): air/water or air/polyethylene oxide solution (PEO), D = 5.15 cm, assumed equivalent length of test pipe = 27 m
Data of Knervold et al. (1984): nitrogen/(kerosene + mineral oil), D = 2.4 cm, assumed equivalent length of test pipe = 30 m.

Effects of pipe diameter and fluid properties

Inspection of Figures 2–4 confirms that Eq. 8 predicts increasing slug frequency with decreasing pipe diameter, as observed empirically. Figure 5 illustrates the calculated effect of pipe diameter on slug frequency for air/water at a fixed liquid superficial velocity. The appearance of maxima in the ω vs. u_{SG} curves predicted by Eq. 8 at low gas rates in the smaller pipes is associated with the laminar-turbulent regime transition of the gas flow in the inlet region. This peculiar feature of slug frequency curves has actually been observed. Taitel and Dukler (1977) reported that Chu's slug frequency data for air/water in

Table 2. Predicted Slug Frequency from Eq. 8 vs. Experimental Data

Data Set	No. of Points	Avg. Error (Hz)	Avg. Abs. Error (Hz)	Avg. Abs. % Error
Dukler and Hubbard (1975)	83	−0.060	0.134	14.2
Heywood and Richardson (1979)	78	−0.088	0.151	16.5
Gregory and Scott (1969)	42	0.088	0.247	16.7
Kago et al. (1987)	14	−0.016	0.044	8.9
Knervold et al. (1984)	5	−0.309	0.309	9.7

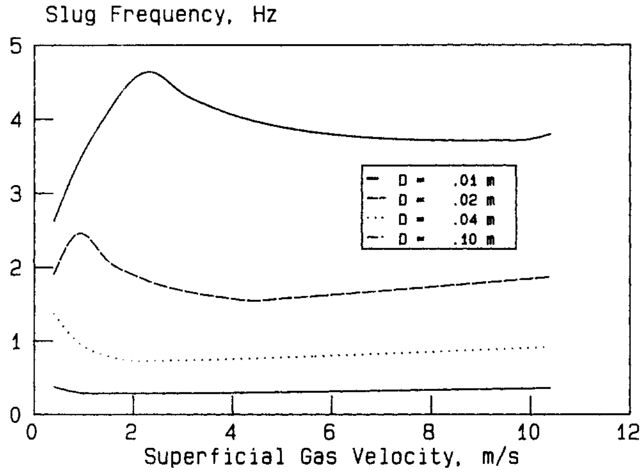

Figure 5. Calculated effect of pipe diameter on slug frequency for intermittent flow of air and water, with $u_{SL} = 0.8$ m/s.
Assumed equivalent length of pipe = 400 diameters.

a 1.9-cm-ID pipe showed "slug suppression" at low gas rates. Likewise, Nicholson et al. (1978) observed the existence of maxima in ω corresponding to very low gas velocities. For example, they mention a sharp peak in the measured values of slug frequency at $u_{SG} = 0.1 - 0.2$ m/s during slug flow in a 5.12-cm-ID pipe at $u_{SL} = 0.12$ m/s. The simulation of these data according to Eq. 8 produces a maximum in ω in the same range of gas flow rates. It should be noted, however, that the capability of Eq. 8 to predict slug frequencies for intermittent flow in very small tubes is possibly limited by the onset of surface tension effects.

Gregory and Scott's data for a CO_2/water system provide some insight into the effects of the gas physical properties on ω. A few of their data are replotted in Figure 6 for a comparison with two different predictions, calculated using the density and viscosity of either CO_2 (curve A) or air (curve B) for the gas phase. In the latter case, a maximum at low gas rates is apparent which is associated with the turbulent-laminar gas transition, as already discussed. With the more dense carbon dioxide, the gas flow is still turbulent even at the lowest experimental gas velocities, and maxima are absent in the calculated slug frequency curves, as displayed by the data.

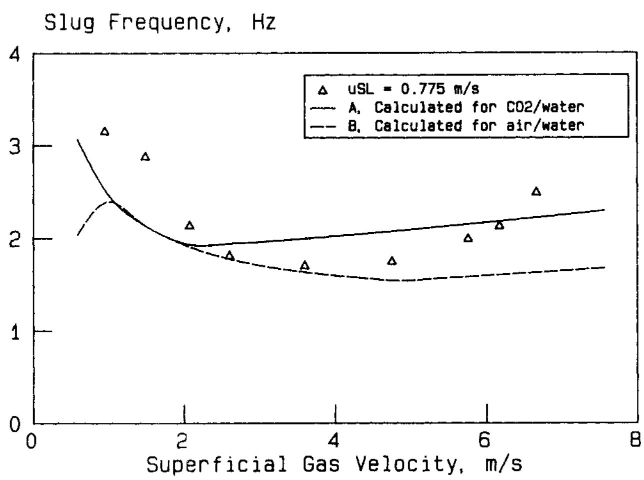

Figure 6. Experimental slug frequency data for CO_2/H_2O (Gregory and Scott, 1969) vs. predictions from Eq. 8 for CO_2/H_2O (curve A) and air/H_2O (curve B).
$u_{SL} = 0.775$ m/s, $D = 1.9$ cm
Assumed equivalent length of test pipe = 5.7 m

Finally, the agreement of theoretical ω-values with the data of Kago et al. (1987) and of Knervold et al. (1984) in Figure 4 indicates that Eq. 8 successfully predicts also the enhancing effect of a greater liquid viscosity on the slug frequency. According to the present theory, such an effect results essentially from the higher equilibrium level of the entry stratified flow. Calculated slug frequencies for three values of the liquid viscosity are shown in Figure 7. The two lower curves indicate that the influence of liquid viscosity is weak when the liquid flow is turbulent in the entry stratified flow: a tenfold increase in viscosity results in only a modest increase in frequency. However, a stronger increment of the frequency occurs on further raising the liquid viscosity to 50 mPa · s, as this brings about the transition to the liquid-laminar regime in the stratified flow at the pipe entrance.

Generalized representation of slug frequency data

If the following dimensionless variables or group of variables are considered,

$$\overline{h}_G = h_G/D; \quad \overline{u}_G = u_G/u_{SG}; \quad \Omega = (\rho_L \omega D/\rho_G u_{SG})$$

Eq. 8 can be rearranged in nondimensional form as

$$\Omega = 0.61\, \overline{u}_G/\overline{h}_G \tag{10}$$

Since both $\overline{u}_G$ and $\overline{h}_G$ are unique functions of the Lockhart-Martinelli parameter X if f_i/f_G = constant, Eq. 10 implies that the dimensionless slug frequency Ω is a function of X only, the functional relationship depending on the flow regimes of the

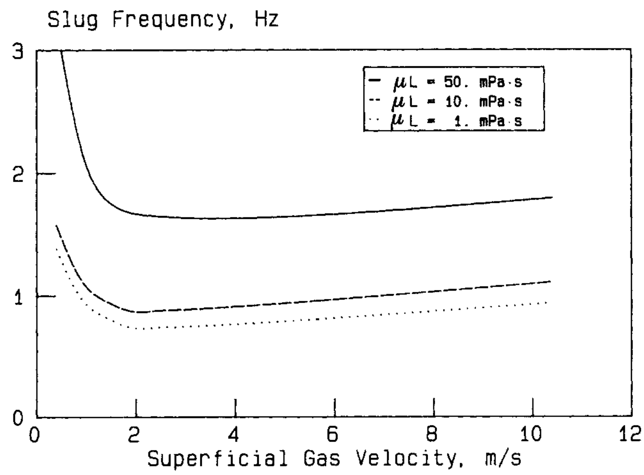

Figure 7. Calculated effect of liquid viscosity on slug frequency at $u_{SL} = 0.8$ m/s in a 4-cm-ID pipe.
Gas phase is air
Liquid density = 1,000 kg/m^3
Assumed equivalent length of pipe = 20 m

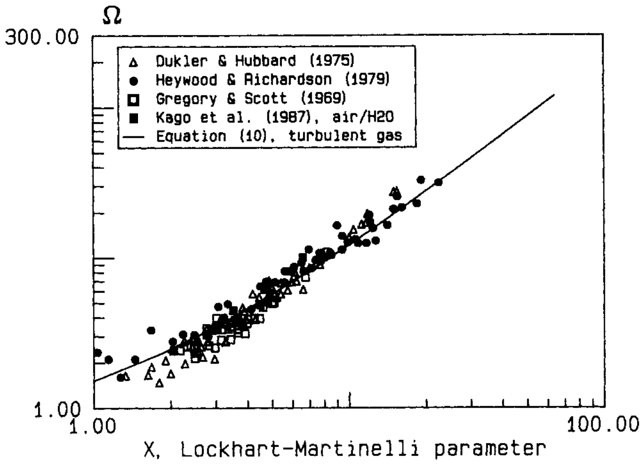

Figure 8. Generalized slug frequency data: dimensionless slug frequency Ω vs. Lockhart-Martinelli parameter *X*.

Case of turbulent gas flow in the stratified flow at the pipe entry
Solid line = predictions of Eq. 10.

liquid and of the gas phase prevailing in the entrance flow. In order to test the validity of this result, the data of Dukler and Hubbard (1975), Heywood and Richardson (1979), Gregory and Scott (1969), and Kago et al. (1987) (limited to air/water) have been plotted as Ω vs. X in Figures 8–9, corresponding to turbulent and transitional + laminar gas flow, respectively. The liquid flow is turbulent in all cases, and the X-values are corrected to pipe entry conditions. The theoretical curves representing Eq. 10 are also displayed for each gas flow regime.

The data points for turbulent gas (Figure 8) fall nicely on the corresponding curve. Diagnostic calculations suggest that a somewhat better agreement could be obtained at high gas rates by using values of $f_i/f_G > 2$, in line with the expected effect of the gas velocity on the interfacial structure. In the absence of reliable expressions for such an influence, however, the assumptions of $f_i/f_G = 2$ seems to result on the whole in an acceptable compromise between accuracy and simplicity.

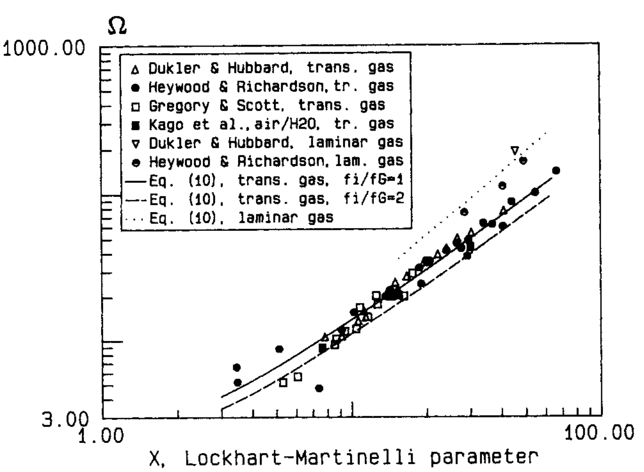

Figure 9. Generalized slug frequency data: dimensionless slug frequency Ω vs. Lockhart-Martinelli parameter *X*.

Cases of transitional and of laminar gas flow in the stratified flow at the pipe entry
Lines = predictions of Eq. 10

For transitional gas flow, Figure 9 shows the curves computed for both $f_i/f_G = 1$ and $f_i/f_G = 2$. Values of Ω calculated using the expression of the interfacial friction factor in Table 1 would fall between those two limits, as indeed do many of the data.

Only four data points corresponding to laminar gas flow are available. These are also remarkably close to the corresponding theoretical curve in Figure 9, so that we conclude that Eq. 10 is suitable for a generalized representation of the available slug frequency data.

Conclusions

The characteristic frequencies of liquid slugs in intermittent gas-liquid horizontal pipe flow was related to the periods of the unstable waves responsible for the generation of slugs in the inlet region of the pipe. The wave properties were estimated in accordance with the theory of finite amplitude waves originally developed by Kordyban and Ranov (1970) and by Mishima and Ishii (1980) to investigate the onset of slugging.

A number of approximations are involved in the present analysis. They derive in part from the simplifying assumptions of the original theory of unstable waves:

1. The Kordyban–Ranov theory was developed for rectangular conduits, but it is applied here to circular pipes.
2. The Kordyban–Ranov theory was developed assuming ideal flow of both gas and liquid, but it is applied here to estimate slug frequencies also when either one of the two phases is in the transitional or laminar regimes.
3. The effects of surface tension are neglected.

Concerning the additional hypothesis that $u_G \approx u_L$, which may also be invalid in practice, a sensitivity analysis has shown that it does not significantly affect the results. Some degree of unavoidable empiricism is also introduced by the assumption of a fixed factor of two between wave frequency and slug frequency. Furthermore, the description adopted for the interfacial shear stress reflects the lack of a satisfactory theoretical treatment of the interfacial structure in stratified wavy flow.

Nevertheless, the comparison of theoretical predictions of ω with data presented in Figures 2–4 and Table 2, and in generalized form in Figures 8–9, indicates that the physical concepts at the origin of the present treatment of slug frequency do not apparently contradict the available experimental observations. On the contrary, they generate predictions in satisfactory agreement with experiment. It is worth emphasizing that the calculated values of ω can reproduce quantitatively and in trend the empirical effects of the main design variables, including flow rates of gas and liquid, pipe diameter, and fluid properties. The maxima and minima unexplained so far in experimental slug-frequency curves are also reproduced.

The small sensitivity to the simplifications of the Kordyban–Ranov theory of waves, and particularly to point 1 above, possibly results from ω being proportional to the ratio of the wave celerity C to the wave length, λ. Thus, if the approximations similarly affect the individual estimates of C and λ, the errors tend to cancel out. The inaccuracies associated with neglecting the surface tension, however, may become significant in the case of small pipes, for which the application of this method is not recommended. Likewise, the overall dependence of the proportionality constant C_1 in Eq. 1 on flow conditions is probably weak due to the contrasting effects of the relevant variables.

For pipe diameters in the range 1.5–5 cm, Eq. 10 enables quick and accurate estimation of the slug frequency for engineering purposes in terms of the usual design variables. In its application, it should be recalled that the Lockhart-Martinelli parameter X must be evaluated at the pipe inlet conditions.

Additional data are required to further validate the present approach. In this respect, valuable insight could be gained from measurements of slug frequency in large diameter pipes with fluids having widely differing properties.

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Notation

A = cross-sectional area of pipe, m^2
C = wave celerity, m/s
C_1 = proportionality constant in Eq. 1
D = pipe diameter, or hydraulic diameter, m
f = friction factor
F = Froude number, $(u^2/gD)^{0.5}$
g = acceleration of gravity, m/s^2
h = liquid level, or gas depth, m
k = wave number, m^{-1}
L = length, m
P = pressure, Pa
Q = function defined in Eq. 7
Re = Reynolds number, $(\rho u D/\mu)$
S = wetted perimeter, m
u = gas or liquid velocity, m/s
x = axial coordinate, m

Greek letters

η = elevation of gas-liquid interface, m
λ = wave length, m
μ = viscosity, Pa · s
ν = wave frequency relative to the moving liquid, s^{-1}
ρ = density, kg/m^3
Φ = functions in Eq. 3
X = Lockhart-Martinelli parameter
ω = slug frequency, s^{-1}
Ω = dimensionless slug frequency

Subscripts and Superscripts

G = gas
I = imaginary part of wave velocity, Eq. 5
i = gas-liquid interface
L = liquid
m, n = exponents in Blasius-type expressions
S = slug
SG = superficial gas
SL = superficial liquid
$-$ = dimensionless variable
$*$ = at the onset of slugging

Literature Cited

Andritsos, N., and T. J. Hanratty, "Influence of Interfacial Waves in Stratified Gas-Liquid Flows," *AIChE J.*, **33**, 444 (1987).

Barnea, D., O. Shoham, and Y. Taitel, "Flow Pattern Transition for Downward Inclined Two Phase Flow: Horizontal to Vertical," *Chem. Eng. Sci.*, **37**, 735 (1982).

Barnea, D., and N. Brauner, "Holdup of the Liquid Slug in Two Phase Intermittent Flow," *Int. J. Multiphase Flow*, **11**, 43 (1985).

Chisolm, D., "A Theoretical Basis for the Lockhart-Martinelli Correlation for Two-Phase Flow," *Int. J. Heat Mass Transfer*, **10**, 1767 (1967).

Cohen, S. L., and T. J. Hanratty, "Effects of Waves at a Gas-Liquid Interface on a Turbulent Air Flow," *J. Fluid Mech.*, **31**, 467 (1968).

Dukler, A. E., and M. G. Hubbard, "A Model for Gas-Liquid Slug Flow in Horizontal and Near Horizontal Tubes," *Ind. Eng. Chem. Fund.*, **14**, 337 (1975).

Dukler, A. E., D. M. Maron, and N. Brauner, "A Physical Model for Predicting the Minimum Stable Slug Length," *Chem. Eng. Sci.*, **40**, 1379 (1985).

Govier, G. W., and K. Aziz, *The Flow of Complex Mixtures in Pipes*, Van Nostrand Reinhold, New York (1972).

Gregory, G. A., and D. S. Scott, "Correlation of Liquid Slug Velocity and Frequency in Horizontal Cocurrent Gas-Liquid Slug Flow," *AIChE J.*, **15**, 933 (1969).

Greskovich, E. J., and A. L. Shrier, "Slug Frequency in Horizontal Gas-Liquid Slug Flow," *Ind. Eng. Chem. Proc Des. Dev.*, **11**, 317 (1972).

Heywood, N. I., and J. F. Richardson, "Slug Flow of Air-Water Mixtures in a Horizontal Pipe: Determination of Liquid Holdup by Γ-Ray Absorption," *Chem. Eng. Sci.*, **34**, 17 (1979).

Hubbard, M. G., "An Analysis of Horizontal Gas-Liquid Flow," PhD Thesis, Univ. of Houston, Houston, TX (1965).

Kago, T., T. Saruwatari, S. Ohno, S. Morooka, and Y. Kato, "Axial Mixing of Liquid in Horizontal Two Phase Slug Flow," *J. Chem. Eng. Japan*, **20**, 252 (1987).

Knervold, O., V. Vindoy, T. Sontvedt, A. Saasen, and S. Selmer-Olsen, "Velocity Distribution in Horizontal Slug Flow," *Int. J. Multiphase Flow*, **10**, 441 (1984).

Kokal, S. L., and J. F. Stanislav, "An Experimental Study of Two-Phase Flow in Slightly Inclined Pipes: I. Flow Patterns," *Chem. Eng. Sci.*, **44**, 665 (1989a).

———, "An Experimental Study of Two-Phase Flow in Slightly Inclined Pipes: II. Liquid Holdup and Pressure Drop," *Chem. Eng. Sci.*, **44**, 681 (1989b).

Kordyban, E., "The Transition to Slug Flow in the Presence of Large Waves," *Int. J. Multiphase Flow*, **3**, 603 (1977a).

———, "Some Characteristics of High Waves in Closed Channels Approaching Kelvin-Helmoltz Instability," *J. Fluid Eng.*, **99**, 339 (1977b).

———, "Some Details of Developing Slugs in Horizontal Two-Phase Flow," *AIChE J.*, **31**, 802 (1985).

Kordyban, E., and T. Ranov, "Mechanism of Slug Formation in Horizontal Two-Phase Slug Flow," *J. Basic. Eng.*, **92**, 857

Lilleleht, L. U., and T. J. Hanratty, "Relation of Interfacial Shear Stress to the Wave Height for Concurrent Air-Water Flow," *AIChE J.*, **7**, 548 (1961).

Lockhart, R. W., and R. C. Martinelli, "Proposed Correlation of Data for Isothermal Two-Phase, Two-Component Flow in Pipes," *Chem. Eng. Prog.*, **45**, 39 (1949).

Maron, D. M., N. Yacoub, and N. Brauner, "New Thoughts on the Mechanism of Gas-Liquid Slug Flow," *Lett., Heat and Mass Transfer*, **9**, 333 (1982).

Mishima, K., and M. Ishii, "Theoretical Prediction of Onset of Horizontal Slug Flow," *J. Fluid Eng.*, **102**, 441 (1980).

Nicholson, M. K., K. Aziz, and G. A. Gregory, "Intermittent Two Phase Flow in Horizontal Pipes: Predictive Models," *Can. J. Chem. Eng.*, **56**, 653 (1978).

Shoham, O., and Y. Taitel, "Stratified Turbulent-Turbulent Gas-Liquid Flow in Horizontal and Inclined Pipes," *AIChE J.*, **30**, 377 (1984).

Taitel, Y., and A. E. Dukler, "A Theoretical Approach to the Lockhart-Martinelli Correlation for Stratified Flow," *Int. J. Multiphase Flow*, **2**, 591 (1976a).

———, "A Model for Predicting Flow Regime Transitions in Horizontal and Near Horizontal Gas-Liquid Flow," *AIChE J.*, **22**, 47 (1976b).

———, "A Model for Slug Frequency During Gas-Liquid Flow in Horizontal and Near Horizontal Pipes," *Int. J. Multiphase Flow*, **3**, 585 (1977).

Vermeulen, L. R., and J. T. Ryan, "Two-Phase Slug Flow in Horizontal and Inclined Tubes," *Can. J. Chem. Eng.*, **49**, 195

Wallis, G. B., and J. E. Dobson, "The Onset of Slugging in Horizontal Stratified Air-Water Flow," *Int. J. Multiphase Flow*, **1**, 173 (1973).

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